International Journal of Theoretical Physics, Vol. 41, No. 6, June 2002 (© 2002)

Equation of State of Hot Neutrino Opaque Interior Matter of Neutron Star

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Received October 12, 2001

The equation of state of hot neutrino opaque interior matter of the neutron star and some of its properties such as the free energy, effective mass, adiabatic index, and temperature are calculated along both isothermal and isentropic paths with the AV_{18} and AV_{14} potentials using the lowest order constrained variational method. We have shown that the calculated equation of state with the AV_{18} potential is harder than with the AV_{14} potential. It is found that there is no phase transition in the hot neutrino opaque interior matter of the neutron star. We have shown that for all values of density and entropy, the adiabatic index of neutron star matter is greater than $\frac{4}{3}$. It is shown that our calculated equations of state obey the causality condition.

KEY WORDS: neutron star matter; equation of state; neutrino opaque; effective mass; adiabatic index.

1. INTRODUCTION

Neutron stars are formed in the gravitational collapse of massive stellar core (Bethe, 1979; Shapiro and Teukolsky, 1983). Just after the formation of a neutron star, it has a hot neutrino opaque interior (Burrows and Lattimer, 1986; Pons *et al.*, 1999; Prakash *et al.*, 1997). The matter in the hot neutrino opaque interior of the neutron star is composed of nucleons (neutrons and protons) and leptons (electrons and positrons) (Burrows and Lattimer, 1986; Pons *et al.*, 1999; Prakash *et al.*, 1997). After complete deleptonization, there is no trapped lepton number, so that neutrinos trapped within the hot interior do not influence the beta equilibrium of neutrons, protons, electrons, and positrons (Burrows and Lattimer, 1986; Pons *et al.*, 1999; Prakash *et al.*, 1997). This is very different from ordinary cold neutron star matter.

The calculation of the equation of state of neutron star matter is the first step in the studies of the properties of this object. It is crucial by important for

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investigating the structure and calculating the mass of neutron star (Heiselberg and Pandharipande, 2000; Lattimer and Prakash, 2000, 2001). Several authors have used different many-body techniques for investigating the properties of neutron star matter, but their results show large differences. These differences become small, if the modern nucleon–nucleon potentials are used (Engvik *et al.*, 1997).

The aim of present work is the calculation of the equation of state of hot neutrino opaque interior matter of the neutron star along both isothermal and isentropic paths. In our calculations, we use the lowest order constrained variational (LOCV) method (Bordbar, 2000, 2001, in press-a, in press-b; Bordbar and Modarres, 1997, 1998; Bordbar and Riazi, 2001a,b; Modarres and Bordbar, 1998) with the modern Argonne potential (AV_{18}) (Wiringa *et al.*, 1995) together with its older model potential (AV_{14}) (Wiringa *et al.*, 1984) for the sake of comparison.

2. NEUTRON STAR MATTER CALCULATIONS

In this paper, we ignore the neutrinos and consider a hot neutron star matter composed of neutrons (n), protons (p), electrons (e^-), and positrons (e^+) in beta equilibrium. Therefore, our neutron star matter calculations divide into two parts. First one is the calculations for nucleonic matter and the other one is the calculations for leptonic matter. We discuss these calculations separately.

2.1. Nucleonic Matter Calculations

As in our previous works, we calculate the energy of nucleonic matter using LOCV method (Bordbar, 2000, 2001, in press-a, in press-b; Bordbar and Modarres, 1997, 1998; Bordbar and Riazi, 2001a,b; Modarres and Bordbar, 1998). We keep only the first two terms in the cluster expansion of the energy functional,

$$E = E_1 + E_2. \tag{1}$$

E_1 is the one-body energy,

$$E_1 = \sum_{i=p,n} \sum_k \frac{\hbar^2 k^2}{2m_i} n_i(k), \tag{2}$$

where $n_i(k)$ is the Fermi–Dirac distribution function (Fetter and Walecka, 1971),

$$n_i(k) = \frac{1}{e^{\beta[\epsilon_i - \mu_i]} + 1}. \tag{3}$$

In the above equation, $\beta = \frac{1}{k_B T}$ (k_B is the Boltzmann constant and T is the temperature), μ_i are the chemical potentials and ϵ_i is defined as

$$\epsilon_i = \frac{\hbar^2 k^2}{2m_i^*}, \tag{4}$$

where m_i^* are the effective masses. For any values of temperature (T) and total number density ($\rho = \rho_n + \rho_p$), the chemical potentials μ_i are determined from the following constraint,

$$\sum_{i=p,n} \sum_k n_i(k) = A. \tag{5}$$

The two-body energy E_2 is

$$E_2 = \frac{1}{2A} \sum_{ij} \langle ij | \nu(12) | ij - ji \rangle, \tag{6}$$

where

$$\nu(12) = -\frac{\hbar^2}{2m}\left[f(12), \left[\nabla_{12}^2,\ f(12)\right]\right] + f(12)V(12)f(12). \tag{7}$$

$f(12)$ and $V(12)$ are the two-body correlation function and nucleon-nucleon potential, respectively.

The procedure for energy calculation of nucleonic matter has also been fully discussed in our previous articles (Bordbar, 2001; Bordbar and Modarres, 1998).

2.2. Leptonic Matter Calculations

The contribution from the energy of electrons and positions is given by

$$E_{\mathrm{L}} = \sum_{i=e^-,e^+} \sum_k n_i(k)[(m_i c^2)^2 + \hbar^2 c^2 k^2]^{\frac{1}{2}}. \tag{8}$$

For electrons and positrons the Fermi–Dirac distribution function $n_i(k)$ is defined as (Fetter and Walecka, 1971)

$$n_i(k) = \frac{1}{e^{\beta[\epsilon_i - \mu_i]} + 1}, \tag{9}$$

where

$$\epsilon_i = [(m_i c^2)^2 + \hbar^2 c^2 k^2]^{\frac{1}{2}}. \tag{10}$$

For a given value of total number density (ρ) and temperature (T), the value of lepton fraction (electron and positron fractions) is determined from the conditions of charge neutrality and beta equilibrium,

$$Y_p = Y_{e^-} - Y_{e^+}, \tag{11}$$

$$\mu_p + \mu_e = \mu_n, \tag{12}$$

while

$$\mu_{e^-} = -\mu_{e^+}. \tag{13}$$

In the above equations, Y_i are the fractions and μ_i are the chemical potentials associated with the neutrons, protons, electrons, and positrons.

3. RESULTS AND DISCUSSION

3.1. Isothermal Case

We have calculated the free energy of hot neutrino opaque interior matter of the neutron star. In our calculations, we have considered the effective masses as variational parameters and minimized the free energy with respect to the variations in them. Our results for the free energy calculations with the AV_{18} and AV_{14} potentials at $T = 15$ and 25 MeV are given in Fig. 1. It is seen that the free energy increases by decreasing temperature. We see that by increasing density, the difference between our results for free energy with the AV_{18} and AV_{14} potentials becomes more appreciable.

The effective masses of the neutron and proton are important in studying the superfluidity in the neutron star matter (Bombaci and Lombardo, 1991). In Fig. 2, we have shown the effective masses of the neutron and proton versus density at $T = 15$ and 25 MeV with the AV_{18} potential. We see that for all values of density, the effective mass of neutron is higher than that of proton. This is due to the

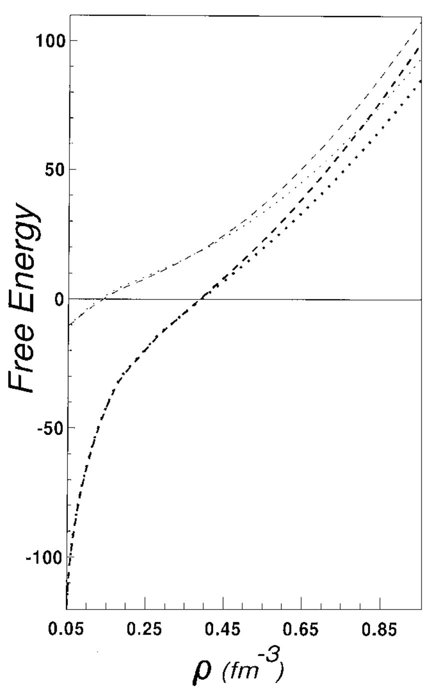

Fig. 1. The free energy versus density for the hot neutrino opaque interior matter of the neutron star with the AV_{18} (at $T = 15$ (dashed curves) and 25 (heavy dashed curves) MeV) and AV_{14} (at $T = 15$ (dotted curves) and 25 (heavy dotted curves) MeV) potentials.

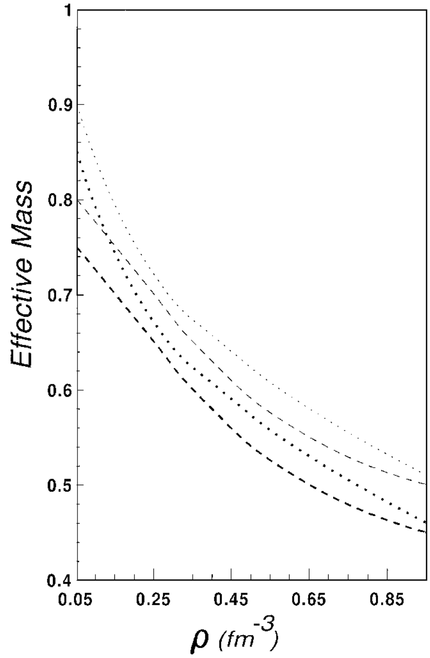

Fig. 2. The effective masses of the neutron (at $T = 15$ (dotted curves) and 25 (dashed curves) MeV) and proton (at $T = 15$ (heavy dotted curves) and 25 (heavy dashed curves) MeV) versus density with the AV_{18} potential.

fact that the proton fraction is low inside neutron star matter. We also see that by increasing both temperature and density, the effective masses of the neutron and proton decrease.

In Fig. 3, we have plotted the equation of state of hot neutrino opaque interior matter of the neutron star with the AV_{18} and AV_{14} potentials at different temperatures ($T = 15$ and 25 MeV). We have found that there is no phase transition in the hot neutrino opaque interior matter of the neutron star. From Fig. 3, we see that for all values of temperature, the equation of state with the AV_{18} potential is stiffer than with the AV_{14} potential.

3.2. Isentropic Case

In the case of constant entropy, we have seen that the internal energy does not vary with the effective mass (Modarres and Bordbar, 1998). Therefore, in our calculations for the isentropic case, we did not vary the effective masses.

The equation of state of hot neutrino opaque interior matter of the neutron star at different values of entropy ($S = 1.0$ and 2.0) with the AV_{18} and AV_{14} potentials are drawn in Fig. 4. As in the isothermal case, it is seen that the isentropic equation of state with the AV_{18} potential is harder than with the AV_{14} potential. It is also seen that at high densities, the differences between the equations of state for different

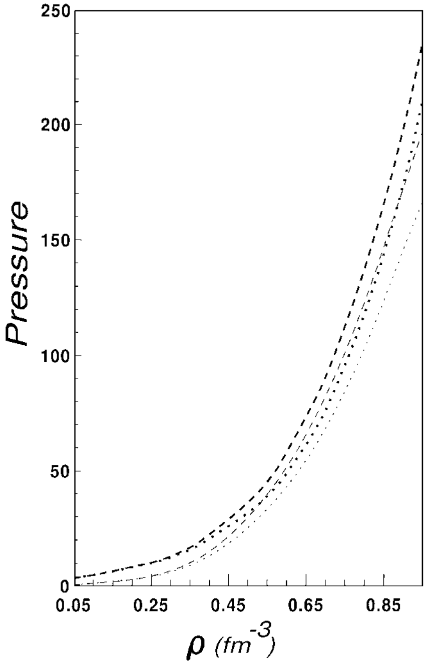

Fig. 3. Same as Fig. 1 but for the equation of state in the isothermal case.

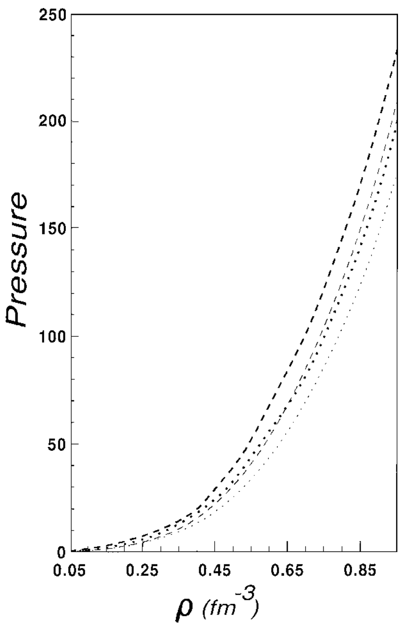

Fig. 4. The isentropic equation of state of hot neutrino opaque interior matter of the neutron star with the AV_{18} (at $S = 1.0$ (dashed curves) and 2.0 (heavy dashed curves)) and AV_{14} (at $S = 1.0$ (dotted curves) and 2.0 (heavy dotted curves)) potentials.

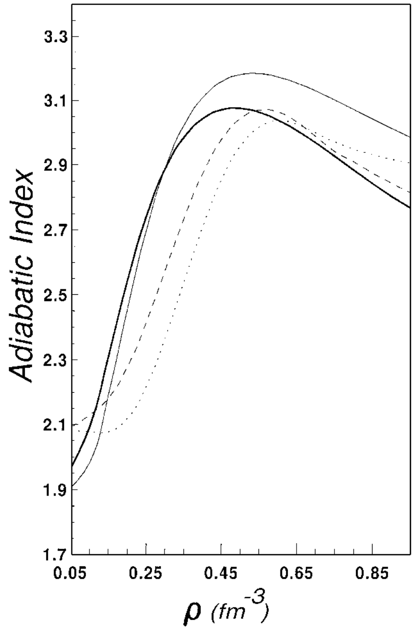

Fig. 5. The adiabatic index versus density for the hot neutrino opaque interior matter of the neutron star at $S = 1.0$ with the AV_{18} (dashed curves) and AV_{14} (dotted curves) potentials. Our results for cold neutron star matter ($S = 0.0$) (Bordbar and Riazi, 2001c) with the AV_{18} (heavy full curves) and AV_{14} (full curves) potentials are given for comparison.

entropies become more considerable. This shows that the equation of state of hot neutrino opaque interior matter of the neutron star is fairly sensitive to the value of entropy.

The adiabatic index Γ plays a crucial role for the stability of neutron star (Shapiro and Teukolsky, 1983). It is given by

$$\Gamma = \frac{\rho}{P}\left(\frac{\partial P}{\partial_\rho}\right)_S . \tag{14}$$

In Fig. 5, we have shown the adiabatic index (Γ) of hot neutrino opaque interior matter of the neutron star with the AV_{18} and AV_{14} potentials at $S = 1.0$. In this figure, we have also shown our results for cold neutron star matter ($S = 0.0$) (Bordbar and Riazi, 2001c). It is seen that the adiabatic index of neutron star matter increases by decreasing entropy. We see that for all values of density and entropy, the adiabatic index of neutron star matter is greater than $\frac{4}{3}$.

In Fig. 6, we have shown the density dependence of temperature for the hot neutrino opaque interior matter of the neutron star with the AV_{18} and AV_{14} potentials at $S = 1.0$ and 2.0. It is seen that the temperature increases by increasing density and decreases by decreasing entropy. It can be seen that the results of the AV_{18} and AV_{14} potentials are nearly identical.

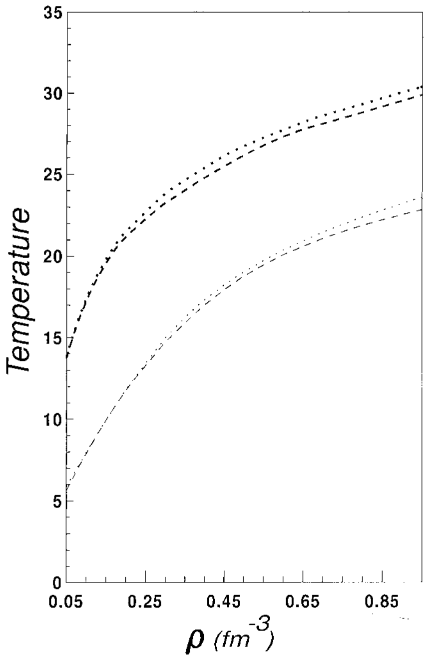

Fig. 6. Same as Fig. 4 but for temperature.

The speed of sound υ_s is given by

$$\upsilon_s = \sqrt{\Gamma \frac{P}{P + \epsilon}}, \tag{15}$$

where ϵ is the mass–energy density,

$$\epsilon = \rho[E + mc^2]. \tag{16}$$

m is the nucleon mass, and c is the speed of light in the vacuum. Our results for υ_s with the AV_{18} and AV_{14} potentials at $S = 1.0$ are given in Fig. 7. We see that the speed of sound increases by increasing density, but it is always smaller than c. This shows that our calculated equations of state obey the causality condition.

4. SUMMARY AND CONCLUSION

In present paper, we have used LOCV method for calculating the equation of state of hot neutrino opaque interior matter of the neutron star and some of its properties for different values of temperature and entropy. In our calculations, we have employed the AV_{18} and AV_{14} potentials. We have considered an uncharged mixture of neutrons, protons, electrons, and positrons in beta equilibrium. It is shown that the free energy of hot neutrino opaque interior matter of the neutron

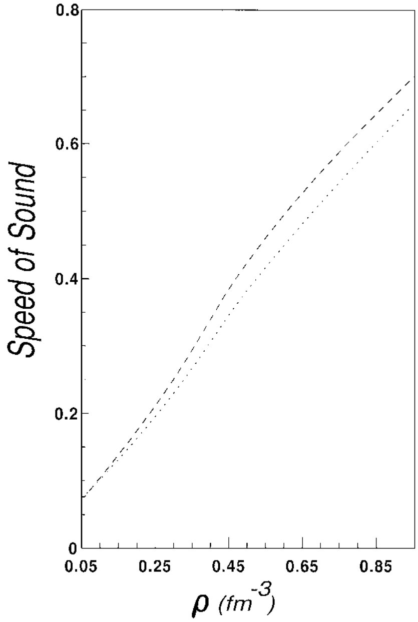

Fig. 7. Speed of sound (in the units of c) versus density at $S = 1.0$ with the AV_{18} (dashed curves) and AV_{14} (dotted curves) potentials.

star increases by decreasing temperature. We have shown that the effective masses of the neutron and proton decrease by increasing both temperature and density. We have also shown that the effective mass of neutron is higher than that of proton. It is seen that for both isothermal and isentropic cases, the equation of state of hot neutrino opaque interior matter of the neutron star with the AV_{18} potential is stiffer than with the AV_{14} potential. Our calculated equations of state have not shown any phase transition in the hot neutrino opaque interior matter of the neutron star. We have seen that the adiabatic index of neutron star matter increases by decreasing entropy. We have also seen that the temperature of hot neutrino opaque interior matter of the neutron star increases by increasing both entropy and density. We have shown that our calculated equations of state of hot neutrino opaque interior matter of the neutron star are causal.

ACKNOWLEDGMENTS

Financial supports from Shiraz University research council and IPM is gratefully acknowledged.

REFERENCES

Bethe, H. A. (1979). *Reviews of Modern Physics* **62**, 801.
Bombaci, I. and Lombardo, U. (1991). *Physics Review C: Nuclear Physics* **44**, 1892.

Bordbar, G. H. and Modarres, M. (1997). *Journal of Physics G: Nuclear and Particle Physics* **23**, 1631.
Bordbar, G. H. and Modarres, M. (1998). *Physics Review C: Nuclear Physics* **57**, 714.
Bordbar, G. H. (2000). *Nuclear Science Journal* **37**, 409.
Bordbar, G. H. (2001). *International Journal of Theoretical Physics G: Theoretical Nonlinear Optics* **7**, 1.
Bordbar, G. H. (in press-a). *International Journal of Theoretical Physics G: Theoretical Nonlinear Optics*.
Bordbar, G. H. (in press-b). *International Journal of Theoretical Physics*.
Bordbar, G. H. and Riazi, N. (2001a). *International Journal of Theoretical Physics G: Theoretical Nonlinear Optics* **7**, 73.
Bordbar, G. H. and Riazi, N. (2001b). *International Journal of Theoretical Physics* **40**, 1.
Bordbar, G. H. and Riazi, N. (2001c). Submitted for publication.
Burrows, A. and Lattimer, J. M. (1986). *Astrophysical Journal* **307**, 178.
Engvik, L., Hjorth-Jensen, M., Machleidt, R., Muther, H., and Polls, A. (1997). *Nuclear Physics A* **627**, 85.
Fetter, A. L. and Walecka, J. D. (1971). *Quantum Theory of Many-Particle Systems*, McGraw-Hill, New York.
Heiselberg, H. and Pandharipande, V. R. (2000). *Annual Review of Nuclear Particle Science* **50**, 481.
Lattimer, J. M. and Prakash, M. (2000). *Physics Report* **333**, 121.
Lattimer, J. M. and Prakash, M. (2001). *Astrophysical Journal* **550**, 426.
Modarres, M. and Bordbar, G. H. (1998). *Physical Review C: Nuclear Physics* **58**, 2781.
Pons, J. A., Reddy, S., Prakash, M., Lattimer, J. M., and Miralles, J. A. (1999). *Astrophysical Journal* **513**, 780.
Prakash, M. *et al.* (1997). *Physical Report* **280**, 1.
Shapiro, S. and Teukolsky, S. (1983). *Black Holes, White Dwarfs and Neutron Stars*, Wiley, New York.
Wiringa, R. B., Smith, R. A., and Ainsworth, T. L. (1984). *Physical Review C: Nuclear Physics* **29**, 1207.
Wiringa, R. B., Stoks, V., and Schiavilla, R. (1995). *Physical Review C: Nuclear Physics* **51**, 38.